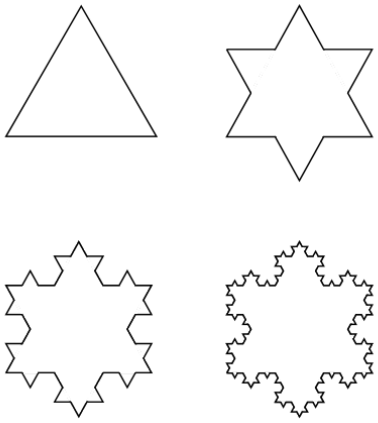
Patterns and Sequences

Hope Chinese School Fall Week 5

September 16, 2017

Problems

1. A 420 page book has an average of 420 words per page. If I read at 490 words per minute, how long, in hours, will I take to finish the book if I don't stop reading?
2. Find $1 - 2 + 3 - 4 + 5 - \cdots + 2015 - 2016 + 2017$.
3. In the sequence of equilateral figures shown, the middle third of each segment is replaced with two segments that are each the same length as the replaced piece. Each side of the first figure (the triangle) is 81 units. What is the perimeter of the fourth figure in the sequence?



4. What is the value of y in the arithmetic sequence $y + 6, 12, y$?
5. One digit of the decimal representation of $\frac{5}{7}$ will be selected at random. What is the probability that the digit will be 4?
6. Find $1 + 3 + 5 + \cdots + 2015 + 2017$.
7. Find the value of
$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\cdots\left(1 - \frac{1}{2017}\right).$$
8. Find the sum of the first forty terms of the sequence $-59, -56, -53, \cdots$.

9. If the pattern shown is continued, what is the sum of the terms in the 12th row?

$$2$$
$$2+4$$
$$2+4+6$$
$$2+4+6+8$$
$$\dots$$

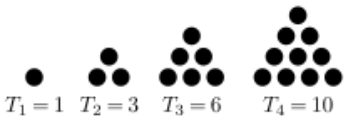
10. Find the sum of all the odd integers between 10 and 50.

11. Find x such that the sequence $4x-1, 2x+2, 2x-3$ is an arithmetic progression.

12. $\star$ Two dogs are running towards each other at 3 meters per second. When they are 60 meters apart, a fly starts traveling between them at 5 meters per second; when it reaches a dog, it turns around and flies towards the other dog. It continues this process until the dogs are next to each other. How far does the fly travel?

Things to know

- Find a pattern!
- If there is lots of symmetry, try to use creative addition.
- Triangular numbers: $T_n = 1+2+\cdots+n = \frac{(n)(n+1)}{2}$.



- Square numbers: n^2.
- Arithmetic sequences: a, $a+d$, $a+2d$, $\dots$, $a+(n-1)d$.