Permutations and Combinations

Hope Chinese School Fall Week 10

October 29, 2017

Do your homework on a **separate sheet of paper.** I will not accept homework done on here. Also, you must provide an explanation for the work you did to get your answer – any answers by themselves do not count.

Each part no longer counts as a full question now. Only full questions count as one question.

1. The honor club, in accordance with the inclusion policy they hastily drafted and passed last Tuesday, is going to accept 97 members from its pool of 100 applicants. How many ways can they select their members?

2. A byte is a sequence of 8 zeros and ones.

   (a) How many different bytes are there in all?

   (b) How many different bytes have 3 ones?

   (c) How many different bytes have 3 zeros?

   (d) How many different bytes start with 3 ones?

3. A local restaurant boasts that they have 240 dinner combinations. A dinner combination consists of an appetizer, entrée and dessert. If the restaurant offers 10 appetizer choices and 6 entrée choices, how many different dessert choices does it have?

4. Cheldelin Middle School has 12 doors to enter or leave the building. In how many ways is it possible to enter the building by one door and leave the building by a different door?

5. A group of 10 people are stuck in the wilderness. They want to make a scouting party of 3 people, with one of them being the leader. How many ways can they choose the scouting party?

6. Consider a regular octagon. How many triangles can be formed whose vertices are the vertices of the octagon?

7. Joe wants to find all the four-letter words that begin and end with the same letter. How many combinations of letters satisfy this property?

8. Robin bought a four-scoop ice cream cone having a scoop each of vanilla, chocolate, strawberry and cherry. In how many orders can the four scoops be stacked on the cone if they are stacked one on top of the other and chocolate cannot be on the bottom?

9. Find and prove the formula for the number of diagonals of a regular n-gon.

10. Prove that $\binom{n}{k} = \binom{n}{n-k}$.

11. ⋆ Ms. Marks is selecting students for a committee that must consist of three seniors and five juniors. Six senior volunteers are able to serve on the committee. What is the least number of junior volunteers needed if Ms. Marks wants at least 100 different possible ways to pick the committee?

12. ⋆ A frog is moving from the point $(0,0)$ to $(5,5)$. If its current location is (x,y), it can hop to the point $(x+1,y)$ or $(x,y+1)$. How many different paths can it take?

13. ⋆ Prove that for all integers n,

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-1} + \binom{n}{n} = 2^n.$$