# Casework and Complementary Counting

Hope Chinese School Fall Week 12

November 11, 2017

1. Alexio has 100 cards numbered 1-100, inclusive, and places them in a box. Alexio then chooses a card from the box at random. What is the probability that the number on the card he chooses is a multiple of 2, 3, or 5?

2. Suppose I have 6 different books, 2 of which are math books. How many ways do I have to stack them on a shelf if I do not want the math books next to each other?

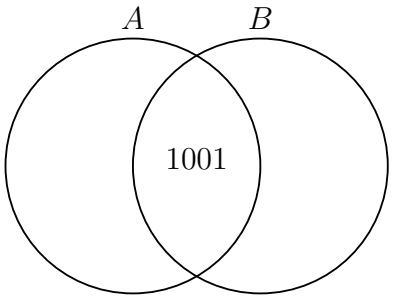
3. How many three digit numbers have no 9s in their digits?

4. There are 8 sprinters in the Olympic 100-meter finals. Three of the sprinters are Americans. The gold medal goes to first place, silver to second, and bronze to third. In how many ways can the medals be awarded if at most one American gets a medal?

5. Every student in the senior class is taking history or science and 85 of them are taking both. If there are 106 seniors taking history and 109 seniors taking science, how many students are in the history class?

6. How many ways are there to put 4 balls in 3 boxes if the balls are not distinguishable but the boxes are?

7. Sets $A$ and $B$, shown in the Venn diagram, have the same number of elements. Thier union has 2007 elements and their intersection has 1001 elements. Find the number of elements in $A$.




8. During a party, each person high-fived everyone else at the party exactly once. If exactly 78 high-fives occurred, how many people were at the party?

9. $\star$ What is the units digit of $1! + 3! + 5! + 7! + 9!$ ?

10. $\star$ 5 people compete in a race. Unfortunately, Alice has rigged the race such that she is guaranteed to finish ahead of Bob. In how many ways different possible orders can the racers finish?