

Probability

Hope Chinese School Fall Week 13

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The probability of something is equal to the number of ways it can occur divided by the total number of possibilities.

Example. *Suppose there are 100 marbles in a bag and 40 of them are green. If I select 1 marble, what is the chance that it is green?*

There are 100 ways to select a marble, and 40 of those ways result in a green marble. Therefore, the probability is $\frac{40}{100} = \frac{2}{5}$.

An important fact of probability is that the probability of two independent events happening is the product of the probability of each event.

Example. *I roll a fair die and flip a fair coin. What is the chance that the die has a 5 and the coin is heads up?*

The die has a $1/6$ chance of rolling a 5, and the coin has a $1/2$ chance of showing heads. The probability that both happen is their product, which is $1/12$. Notice that this is very related to multiplication in counting: there are 6 possibilities for the die and 2 for the coin, so there are $6 \times 2 = 12$ in all. Exactly 1 of them is “good”.

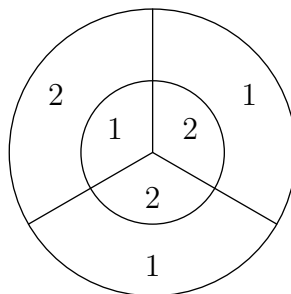
Also, the probability of all possible events is equal to 1. We can use this to find probability algebraically.

Example. *A rigged coin shows heads twice as often as it comes up tails. What is the probability that it comes up heads?*

Let the probability of heads on the coin be H , and the probability of tails be T . First of all, the only possible events are heads and tails, so we know that $H + T = 1$. We can also translate the first sentence of the problem to say $H = 2T$. Solving, we get $H = 2/3$.

1. A club has 20 members, 12 boys and 8 girls. Two of the members are chosen at random. What is the probability that a boy and a girl are chosen?
2. 3 red balls and 5 blue balls are placed in a bag. Nancy randomly grabs 2 of them from the bag.
 - (a) What is the probability that they are both red?
 - (b) What is the probability that they are different colors?
3. Keiko tosses one penny and Ephraim tosses two pennies. What is the probability that Ephraim gets the same number of heads that Keiko gets? Express your answer as a common fraction.

4. I roll 3 fair dice. What is the probability that none of them show a 5?
5. In a history class, the probability of earning an A is .7 times the probability of earning a B, and the probability of earning a C is 1.4 times the probability of earning a B. Assuming that all grades are A, B, or C, how many B's will there be in a history class of 31 students?
6. At breakfast, lunch, and dinner, Joe randomly chooses with equal probabilities either an apple, an orange, or a banana to eat. On a given day, what is the probability that Joe will eat at least two different kinds of fruit?
7. If I roll 4 fair dice and multiply the numbers shown on the faces together, what is the probability that I get an even number? What about a prime number?
8. Eight women of different heights are at a party. Each woman decides to only shake hands with women shorter than herself. How many handshakes take place?
9. On the dart board shown in the figure, the outer circle has radius 6 and the inner circle has radius 3. Three radii divide each circle into the three congruent regions, with point values shown. The probability that a dart will hit a given region is proportional to the area of the region. What two darts hit this board, the score is the sum of the point values in the regions. What is the probability that the score is odd?



10. ★ A rigged coin shows heads 3 times as often as tails. If I flip it 4 times, what is the chance that I get 3 heads and 1 tail?
11. ★ (Birthday Paradox) At least how many people need to be in a room such that there is more than one half chance that *any* two people share the same birthday?