

Remainders

Hope Chinese School Fall Week 14

December 2, 2017

Problems

1. What is the remainder when 2017 is divided by 80?
2. A bunch of cookies are to be evenly distributed to 10 people desperately in need such that each person gets an equal number of cookies, and the cookies that are left over will be given to the Cookie Monster. If the 10 people each get the greatest possible number of cookies, at most how many cookies will the Cookie Monster get?
3. How many positive two-digit integers leave a remainder of 2 when divided by 8?
4. What is the remainder when $(99)(101)$ is divided by 9?
5. In this problem, a and b are integers.
If $a + b \equiv 2 \pmod{10}$ and $2a + b \equiv 1 \pmod{10}$, then what is the last digit of $a - b$?
6. What is the remainder when $2000 + 2001 + 2002 + 2003 + 2004 + 2005 + 2006$ is divided by 7?
7. What is the last digit of $1! + 2! + 3! + \cdots + 20!$?
8. What is the remainder when $2^{87} + 3$ is divided by 7?
9. ★ A positive integer N satisfies $N \equiv 1 \pmod{2}$, $N \equiv 2 \pmod{3}$, $N \equiv 3 \pmod{4}$, \dots , $N \equiv 9 \pmod{10}$. What is the smallest possible positive value of N ?
10. ★ For a prime p , show that if Alice picks an integer a such that p does not divide a , Bob can always pick an integer b such that ab leaves a remainder of 1 when divided by p . (Bonus: show this isn't true when p is composite.)