

# Quadratic Equations

Hope Chinese School Spring Week 18

January 20, 2018

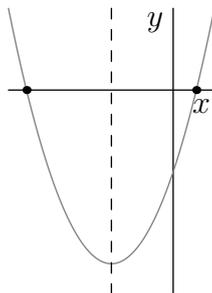
Remember your manipulation skills from last week! They will come in handy here.

1. A *cubic* polynomial is similar to a quadratic, except that it has an  $x^3$  term as its largest. They have the form  $ax^3 + bx^2 + cx + d$ .
  - (a) In terms of  $a, b, c$ , or  $d$ , what is the sum of the roots of a cubic?
  - (b) What about the product?
  - (c) What do you think  $c/a$  represents (in terms of the roots)?
2. There are two values of  $a$  for which the equation  $4x^2 + ax + 8x + 9 = 0$  has only one solution for  $x$ . What is the sum of these values of  $a$ ?
3. A quadratic equation  $ax^2 - 2ax + b = 0$  has two real solutions. What is the average of the solutions?
4. What is the greatest product obtainable from two integers whose sum is 246?
5. What is the maximum value of  $4(x + 7)(2 - x)$  for all real values  $x$ ?
6. Let  $a$  and  $b$  be the roots of  $x^2 - 2018x + 50$ . What is  $(a - 1)(b - 1)$ ?
7. The quadratic equation  $x^2 + mx + n = 0$  has roots that are twice those of  $x^2 + px + m = 0$ , and none of  $m, n$ , and  $p$  is zero. What is the value of  $\frac{n}{p}$ ?
8. A monic polynomial<sup>1</sup>  $P$  has roots 1, 2, 3, 4, 5, 6, 7, 42, 1337. Find the sum of the coefficients of  $P$ .
9. ★ What is the sum of the reciprocals of the roots of the equation

$$\frac{2003}{2004}x + 1 + \frac{1}{x} = 0?$$

10. ★ Prove the two-variable AM-GM inequality: for any nonnegative reals  $a, b$ ,

$$\frac{a + b}{2} \geq \sqrt{ab}.$$



### Optimization of Quadratics

An important symmetry of all quadratic functions on the reals is a vertical line of symmetry cutting through its minimum/maximum.

1. One consequence of this symmetry is that the roots are equidistant from the line of symmetry. Use this to find the real value of  $x$ , in terms of  $a, b, c$ , that minimizes  $ax^2 + bx + c$ , if  $a > 0$ .

(Hint: this means that this value of  $x$  is the average of the roots.)

2. Sometimes you see people write quadratics as  $a(x - h)^2 + k$ .

- (a) Why is  $k$  the minimum (maximum if  $a < 0$ ) value of the quadratic, and why does  $x = h$  generate that value?

(Hint: what is the minimum value of  $(x - h)^2$ ?)

- (b) Now find this  $h$  and  $k$  for a quadratic  $ax^2 + bx + c$ .

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<sup>1</sup>Monic polynomial: a polynomial with leading coefficient 1.