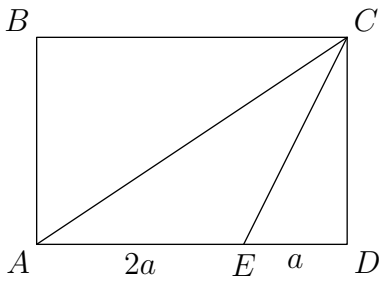
Areas, Lengths, and Ratios

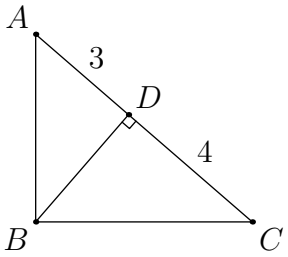
Hope Chinese School Spring Week 20

February 3, 2018

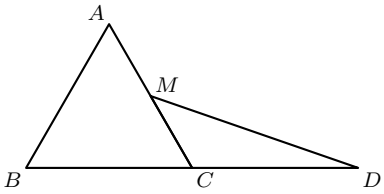
1. Find the ratio of the area of $\triangle ACE$ to the area of rectangle $ABCD$.



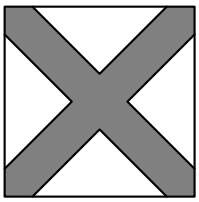
2. Triangle ABC has a right angle at B. Point D is the foot of the altitude from B, $AD = 3$, and $DC = 4$. What is the area of $\triangle ABC$?



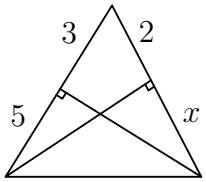
3. Equilateral $\triangle ABC$ has side length 2, M is the midpoint of $\overline{AC}$, and C is the midpoint of $\overline{BD}$. What is the area of $\triangle CDM$?



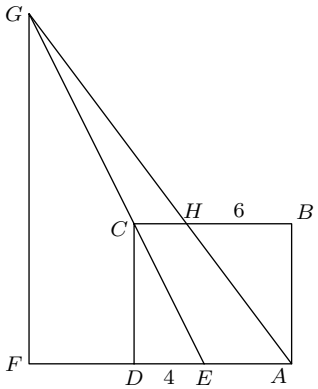
4. A paint brush is swept along both diagonals of a square to produce the symmetric painted area, as shown. Half the area of the square is painted. What is the ratio of the side length of the square to the brush width?



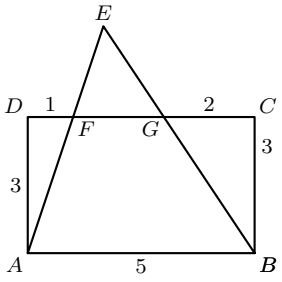
5. Two of the altitudes of an acute triangle divide the sides into segments of lengths $5, 3, 2$ and x units, as shown. What is the value of x?



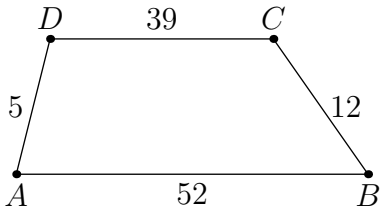
6. In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, H is on $\overline{BC}$ with $BH = 6$, E is on $\overline{AD}$ with $DE = 4$, line EC intersects line AH at G, and F is on line AD with $\overline{GF} \perp \overline{AF}$. Find the length GF.



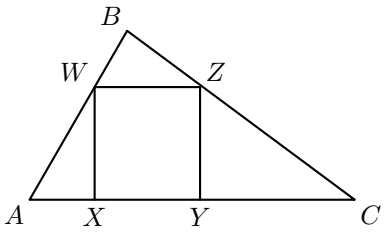
7. In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on $\overline{CD}$ so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E. Find the area of $\triangle AEB$.



8. $\star$ In trapezoid $ABCD$ with bases AB and CD, we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. Find the area of $ABCD$.



9. ⋆ Right $\triangle ABC$ has $AB = 3$, $BC = 4$, and $AC = 5$. Square $XYZW$ is inscribed in $\triangle ABC$ with X and Y on $\overline{AC}$, W on $\overline{AB}$, and Z on $\overline{BC}$. What is the side length of the square?



A few tips (just in case)

- *The information game involves hunting for lengths and angles.*
- **Look for similar triangles.** Similar triangles are the most powerful tool at your disposal, with the massive amount of length and angle information they carry.
- Right triangles are quite nice, but only use Pythagorean theorem when necessary. However, if you do see a Pythagorean triple, that usually means you are making massive progress.
- For all triangles, $A = bh/2$. **Do not underestimate the power of this fact!** It is the primary way allowing you to exploit areas by connecting to lengths. Keep in mind, this $bh/2$ is constant no matter which base you are using; this gives you length information!
- In some diagrams with parallel lines, connect the other lines: this will form several similar triangles.